On the Correlation Radiometer Technique

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Summary—**The correlation radiometer is a system in which two antenna-receiver systems are employed, the outputs of the two systems being correlated electronically. This paper studies the two basic types of correlation radiometers, the intermediate-frequency correlation (IF) type and the video-frequency correlation (ENV) type. The SNR's, the minimum detectable temperature sensitivities, the effects of receiver gain and phase fluctuations and the uses of the two types are studied. A comparison of the various characteristics of the two types of correlation radiometers with the same characteristics of the Dicke-type radiometer is made.**

Introduction to the Correlation Radiometer

THE MOST IMPORTANT problem in radiometer studies is to establish the lowest detectable source temperature. The minimum detectable temperature is usually determined by the noise fluctuations appearing in the receiver output. Since the source temperature is measured in the form of thermal radiation from the source, special techniques must be employed to reduce the spurious fluctuations in the output produced by the receiver circuits and to differentiate these from the real signal. The conventional method for reducing the spurious spectrum and noise effects of the receiver is to employ an optimum modulation of signal so that the spurious spectrum and noise are cancelled out, as in the well-known Dicke type of system [1].

Many radiometer types have been investigated previously, and theoretical as well as practical studies have been performed for both microwave and millimeter-wave applications of radiometers [1]–[18]. However, the most commonly used one is still the Dicke-type radiometer and its various modified versions. Correlation technique [19]–[21], [23] and its application to radiometers also have been discussed in several previous works [3], [11], [22].

It is the purpose of this paper to unify the theory which, to a large extent, already has been developed in these references, and to present it in such a form that a systematic comparison between the correlation-type and Dicke-type radiometers can be made.

The correlation radiometer consists of two receiver systems with separate antennas, as shown in Fig. 1. Both antennas are looking at the same signal source; thus the two signals s_1 and s_2 will be correlated in time, and upon multiplication they will provide an output proportional to the source signal strength s. The noise n_1 and n_2 introduced by each receiver will necessarily have a low degree of correlation because of the random nature of n_1 and n_2; thus the total correlated output will represent the signal s plus some low level of correlation between n_1 and n_2. In other words, by the use of correlation techniques, the sensitivity of the radiometer may be greatly increased as a consequence of the low degree of correlation of n_1 and n_2. Although the two signals are considered to be coherent in both phase and amplitude to the first approximation, in practice one cannot expect perfect coherence since one or the other of the signals may be perturbed by the medium through which the signals are passing on their way from the source to the radiometer. In some cases the phase coherence is almost totally destroyed and only the amplitudes of the two signals retain any degree of correlation. In order to use correlation techniques in this case, one must employ square-law envelope detection before the correlation process, as is shown in Fig. 2. We distinguish between the two types of radiometers by calling the first type (where the signals themselves are correlated) the IF type, and the second type (where only the amplitudes are correlated) the ENVELOPE type.

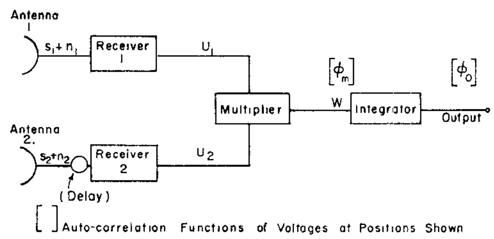

Fig. 1—Simple block diagram for the IF type of correlation radiometer.

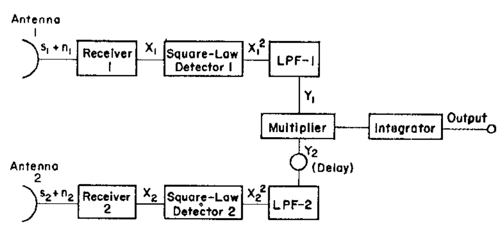

Fig. 2—Simple block diagram for the ENV type of correlation radiometer.

The correlation radiometer has the same order of sensitivity as the Dicke type. Some of the advantages and disadvantages of the correlation type of radiometer have been discussed by other authors [3], [11], [22] and are tabulated below.

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Disadvantages of Correlation Radiometer

1) Because of the need for two identical receivers, the correlation radiometer systems are necessarily more complex.

2) Fluctuations in the gain or phase characteristics of the receivers in the correlation radiometer will cause greater output fluctuations than in the case of the Dicke radiometer, where the temperature of the matched load may be made almost the same as the apparent source temperature to reduce the effects of gain fluctuations. Also, phase shifts in the Dicke radiometer receiver have little or no effect upon the output while, in the case of the IF type of correlation radiometer, spurious phase shifts in either one of the two receivers will seriously degrade the performance of the radiometer.

Advantages of Correlation Radiometer

1) The correlation radiometer requires no switching scheme, as does the Dicke type. This is especially important at the shorter wavelengths where a microwave or optical switch would be more complex and lossy than a similar switch at the longer microwave wavelengths.

2) The IF type of correlation radiometer is easily adapted for use in an interferometer system. Using this type of system, the steering of the antenna beam can often be accomplished by means of phase-shift techniques without the need for the mechanical movement of the radiometer antenna.

3) The correlation radiometer (especially the ENV type) can be used for the simultaneous observation of two signals and the determination of the correlation existing between them. For example, one can use the ENV correlation technique in the analysis of the scattering or reflection characteristics of the moon (Fig. 3). In this application one antenna would observe the sun (antenna 1) and the second would observe the moon (antenna 2). At certain wavelengths the reflected solar signal from the moon might be quite weak as compared with the background radiation of the moon, due to its own temperature. If the Dicke type of system is employed, both the reflected and the background radiation are chopped, and hence it would be quite difficult to distinguish between the two types of radiation. However, in the correlation radiometer the reflected radiation would be highly correlated (in amplitude) with the direct radiation from the sun, while the background radiation from the moon would not. Thus, the amount of total radiation from the moon which is due to reflection may be determined. It might be necessary, however, to introduce a time delay in one of the receiver channels to account for the path-length difference between the direct path from the sun to the earth as compared to the path from the sun to the earth via the moon.

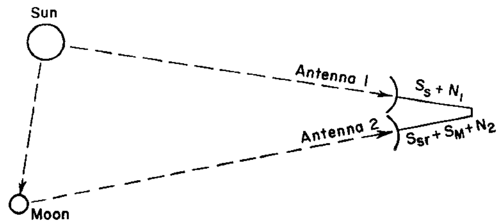

Fig. 3—A proposed experiment to study the bistatic reflection from the moon using a correlation radiometer.

In this paper, expressions for the SNR and the minimum detectable temperature will be determined for both the IF and ENV types of correlation radiometers and these will be compared with similar equations for the Dicke radiometer.

Analysis of the Correlation Radiometer

The Receiver Signals

In order to evaluate the minimum detectable signal in terms of the equivalent minimum detectable temperature increment (ΔT) or the minimum detectable temperature $T_{\min}$, one must evaluate the SNR at the output of the radiometer system in terms of the SNR at the input to the radiometer receivers. We first assume the following:

1) Both the signal and noise possess Gaussian distributions with zero mean values.
2) Both functions are independent of other variables, and they have the property of ergodicity and are wide-sense stationary.
3) The signals at the receiver inputs are coherent and each has a mean-square value ψ_{sj}.
4) The noise signals at the receiver inputs are uncorrelated and each has a mean-square value ψ_{nj}.

(The subscripts $j=1$, 2 denote the channel or receiver.)

Basic Relations Pertaining to the Correlation of Two Signals

Referring to Fig. 1, the inputs at the correlator are represented by $U_1(t)$ and $U_2(t+\theta)$, where θ is the time delay of channel 1 as compared to channel 2. The correlator circuit consists of a multiplier followed by an integrator circuit. The output of the multiplier has the correlation function

$$\phi_{m(\tau,\theta)} = \overline{W_{(t,\theta)} \cdot W_{(t+\tau,\theta)}}, \tag{1}$$

where

$$W_{(t,\theta)} = U_{1(t)} \cdot U_{2(t+\theta)}. \tag{2}$$

(The overhead bar denotes a time average.)

The portion of $\phi_{m(\tau,\theta)}$ which is independent of τ (*i.e.*, $\phi_{m(\theta)}]_{dc}$) represents the desired signal power output and the portion which is dependent upon τ but has no dc component represents the noise power output of the multiplier circuit. The action of the integrator part of the correlation serves to reduce this latter portion in the correlator output, and in general, the longer the integration time, the lower will be the τ-dependent portion of the correlator output as compared to the dc portion. The output of the integrator can be found by convolving the integrator input with its impulse response function. Taking the ratio of the dc to the τ-dependent portions of the correlator output then gives the output power SNR, which for $\theta=0$ is[1]

$$\frac{S}{N}(\tau=0,\theta=0) = \frac{\phi_m(0)]_{dc}}{\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\phi_{m(\tau',0)}]_{ac}\,|H(\omega)|^2\epsilon^{-j\omega\tau'}d\omega d\tau'}, \quad (3)$$

where $H(\omega)$ is the transfer function of the integrator circuit. The sensitivity in terms of the minimum detectable equivalent temperature is defined as that which would give a unity SNR at the output. An expression for (ΔT) or $T_{\min}$ will be derived later on.

The above has considered the IF type of correlation radiometer. Completely similar relations can be derived for the ENV type if U_1 and U_2 are replaced by Y_1 and Y_2 as shown in Fig. 2.

Signal-to-Noise Ratio

IF type of correlator: Referring to Fig. 1, the input signal to the correlator (*i.e.*, the receiver output signal) for each channel is given by

$$U_{j(t)} = A_{j(t)}\{s_{j(t)} + n_{j(t)}\}, \quad (4)$$

where $s_{j(t)}$ is the input signal voltage and $n_{j(t)}$ is the equivalent receiver input noise voltage. $A_{j(t)}$ is the gain of receiver, which in the initial analysis will be taken as constant (*i.e.*, $A_{j(t)}=A_{0j}$). The following relations between the two input signals to the receiver are assumed:

$$s_{1(t)} = s_{(t)}, \quad (5a)$$

$$s_{2(t)} = \eta s_{(t+\theta)}, \quad (5b)$$

where η is the amplitude factor and θ is the relative time delay of the channel 1 signal as compared to the channel 2 signal. When the two antennas are looking at the same source and the separation between the two antennas is small, $\eta \doteq 1$. We assume that the autocorrelation function of both signal and noise after passing through high-Q bandpass-type amplifier has the form [24]

$$\phi_s = \psi_s\epsilon^{-\omega_s|\tau|}\cos\omega_0\tau, \quad (6a)$$

$$\phi_n = \psi_n\epsilon^{-\omega_n|\tau|}\cos\omega_0\tau, \quad (6b)$$

where ω_0 is the center frequency of the amplifier and ω_s and ω_n are the effective half-bandwidth of signal and noise, respectively. The dc component of (1) is found to be

$$\phi_{m(\theta)}]_{dc} = \eta^2\psi_s^2\epsilon^{-2\omega_s\theta}(\cos\omega_0\theta)^2. \quad (7)$$

If we assume that the transfer function of the integrator circuit is

$$H(\omega) = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_L}\right)^2}}, \quad (8)$$

where ω_L is the angular cut off frequency of the integrator, then the output *noise* can be shown to have the following correlation function at $\tau=0$:

$$\phi_{0(0,\theta)} \doteq \frac{\omega_L}{2}\Bigg[\eta^2\psi_s^2\left\{\frac{1}{2\omega_s}+\left(\frac{1-\epsilon^{-2\omega_L\theta}}{\omega_L}+\frac{1}{2\omega_s}\right)\cdot\epsilon^{-2\omega_s\theta}\cos 2\omega_0\theta\right\} + \psi_s\psi_{n2}\left(\frac{1}{\omega_s+\omega_{n2}}\right) + \eta^2\psi_s\psi_{n1}\left(\frac{1}{\omega_s+\omega_{n1}}\right) + \psi_{n1}\psi_{n2}\left(\frac{1}{\omega_{n1}+\omega_{n2}}\right)\Bigg] \quad (9$$

where terms above the second order have been omitted. Thus the output power SNR can be written as (for $\theta=0$)

$$\frac{S}{N}\bigg]_I = \frac{2\eta^2\psi_s^2\left(\frac{\Delta\omega_{IF}}{\omega_L}\right)}{2\eta^2\psi_s^2+\eta^2\psi_s\psi_{n1}+\psi_s\psi_{n2}+\psi_{n1}\psi_{n2}} \quad (10)$$

where $\omega_s=\omega_{nj}=\Delta\omega_{IF}/2$ has been assumed (the I subscript refers to IF-type correlation, E will be used for ENV type). From (4) and (5) we can find the SNR at the correlator input to be

$$\frac{S}{N}\bigg]_{in,1} = \underset{(\tau=0)}{R_{in,1}} = \frac{\overline{s_{1(t)}s_{1(t+t)}}}{\overline{n_{1(t)}n_{1(t+\tau)}}}\bigg]_{\tau=0} = \frac{\phi_{s(\tau)}}{\phi_{n1(\tau)}}\bigg]_{\tau=0} = \frac{\psi_s}{\psi_{n1}} = R_1 \quad (11)$$

[1] Eq. (3) and many other equations presented in this paper are derived in K. Fujimoto, "On the Correlation Radiometer Technique, II," Antenna Lab., The Ohio State University, Columbus, Rept. No. 1093-16; 1963.

for channel 1, and

$$\left.\frac{S}{N}\right]_{\text{in},2} = \underset{(\tau=0)}{R_{\text{in},2}} = \left.\frac{\overline{s_{2(t)}s_{2(t+\tau)}}}{\overline{n_{2(t)}n_{2(t+\tau)}}}\right]_{\tau=0} = \left.\frac{\eta^2\phi_{s(\tau)}}{\phi_{n2(\tau)}}\right]_{\tau=0} = \frac{\eta^2\psi_s}{\psi_{n2}} = \eta^2 R_2 \tag{12}$$

for channel 2. Using the above, it can be shown that the SNR at the correlator output for the IF case then becomes

$$\left.\frac{S}{N}\right]_I = R_I = \frac{2R_{\text{in},1}, R_{\text{in},2}}{(R_{\text{in},1})(R_{\text{in},2}) + (1 + R_{\text{in},1})(1 + R_{\text{in},2})}\cdot\alpha \tag{13a}$$

or

$$R_I = \frac{2\eta^2 R_1 R_2}{(\eta^2 R_1 R_2) + (1 + R_1)(1 + \eta^2 R_2)}\cdot\alpha, \tag{13b}$$

when expressed as a function of the input SNR, where $\alpha = \Delta\omega_{IF}/\omega_L$.

ENV type of correlator: The output power SNR for this type of radiometer can be found by the use of the above-developed equations where instead of $R_{\text{in},1}$ and $R_{\text{in},2}$ we make use of two new quantities

$$R'_{\text{in},1} = \frac{\psi_s^2}{2\psi_s\psi_{n1} + \psi_{n1}^2} = \frac{(R_{\text{in},1})^2}{(2R_{\text{in},1} + 1)} = \frac{R_1^2}{(2R_1 + 1)} \tag{14}$$

and

$$R'_{\text{in},2} = \frac{\eta^4\psi_s^2}{2\eta^2\psi_s\psi_{n2} + \psi_{n2}^2} = \frac{(R_{\text{in},2})^2}{(2R_{\text{in},2} + 1)} = \frac{\eta^4 R_2^2}{(2\eta^2 R_2 + 1)} \tag{15}$$

to define the SNR at the correlator input in the ENV type of system.[2] This leads to the following expression for the SNR at the output of the correlator:

$$\left.\frac{S}{N}\right]_E = R_E = \frac{2(R_{\text{in},1})^2(R_{\text{in},2})^2}{(R_{\text{in},1})^2(R_{\text{in},2})^2 + (1 + R_{\text{in},1})^2(1 + R_{\text{in},2})^2}\cdot\beta, \tag{16a}$$

or

$$R_E = \frac{2\eta^4(R_1)^2(R_2)^2}{\eta^4 R_1^2 R_2^2 + (1 + R_1)^2(1 + \eta^2 R_2)^2}\cdot\beta, \tag{16b}$$

where $\beta = \Delta\omega_d/\omega_L$ ($\Delta\omega_d$ is the bandwidth immediately preceding the correlator). From the preceding, one can see that $R'_{\text{in},j}$ is always smaller than $R_{\text{in},j}$ and at most $R'_{\text{in},j} = \frac{1}{2}R_{\text{in},j}(\eta \leq 1)$. In other words, from the standpoint of the SNR, placing square-law detectors before the multiplier causes a loss in the radiometer sensitivity, especially in the case where the receiver input SNR is much less than unity. Thus, the use of the ENV type of correlation radiometer should be avoided except in those cases where the IF type is not practical (*i.e.*, where there is no phase coherence between the incoming signals to the two receivers). The relation between $R'_{\text{in},j}$ and R_j is shown in Fig. 4, and plots of R_I and R_E vs R_1 for various values of $\eta(\leq 1)$ are shown in Figs. 5 and 6, respectively. These plots assume the condition that $\psi_{n1} = \psi_{n2}$, or $R_1 = R_2$. The solid lines in Figs. 4–6 show the unity locus where the input and the output SNR's are equal. In the ENV type of system the degradation of the output SNR occurs more rapidly than it does in the IF type of system, as the input SNR becomes smaller. However, in either case, since α and β are usually much greater than unity, we find that one can obtain a much greater output SNR than input SNR.

[2] Eqs. (14) and (15) are the results obtained by the use of the limiting values of the autocorrelation functions $\rho(\tau)_{\max} = 1$ for signal and noise voltages (envelope).

The Minimum Detectable Temperature Increment

The effect of the input sinal strength: If the output SNR is near unity (as, for example, when one or both of the input SNR's are small), then it can be shown that the minimum detectable temperature $T_{\min}$ (which can be detected by the system) is proportional to ψ_s, which is the mean-square value of the receiver input signal.

If now it is assumed that the output SNR is large, then (ΔT) represents the minimum incremental variation in the temperature of the source which can be detected and replaces $T_{\min}$ as the measure of sensitivity. We define (ΔT) as the change in the apparent source temperature which produces a change in the correlator output equal to the rms fluctuations in the output noise level. If we let S be the output signal power of the radiometer, then, in general,

$$S = c_I T_1 T_2 \qquad \text{(IF type)}, \tag{17}$$

or

$$S = c_E T_1^2 T_2^2 \qquad \text{(ENV type)}, \tag{18}$$

where c_I and c_E are constants. Differentiating (17) and (18) and assuming that $\Delta T_2 = \eta^2 \Delta T_1$ and $T_2 = \eta^2 T_1$, gives

$$\Delta S/S = 2(\Delta T_1)/T_1 \qquad \text{(IF type)} \tag{19}$$

or

$$\Delta S/S = 4(\Delta T_1)/T_1 \qquad \text{(ENV type)}. \tag{20}$$

But in view of our definition of (ΔT), the fluctuations ΔS in the output must be equal to the noise power N, so that

$$\Delta T_1 = T_1(\Delta S)/\xi S = T_1 N/\xi S, \tag{21}$$

where $\xi = 2$ or 4 according to whether the IF type or the ENV type of system is being considered. Here ΔT_1 is understood to be equal to (ΔT).

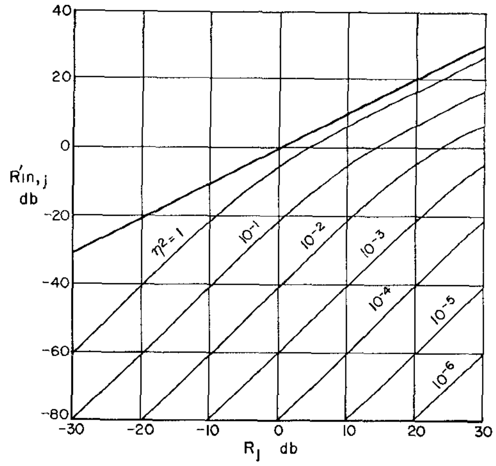

Fig. 4—The correlator input SNR as a function of the receiver input SNR (for $j=1$, $\eta^2=1$).

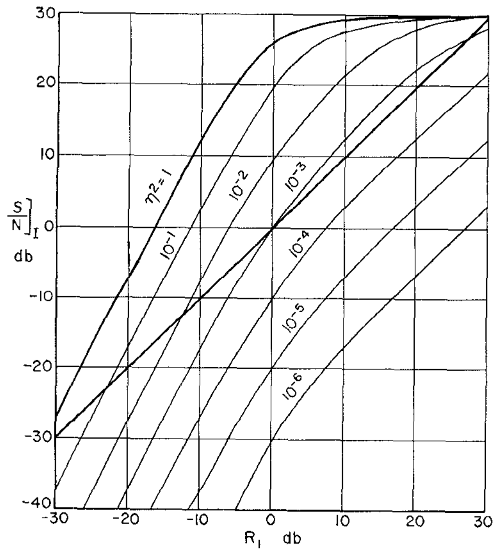

Fig. 5—The output SNR for the IF type of correlation radiometer as a function of the receiver input SNR ($\alpha=10^3$).

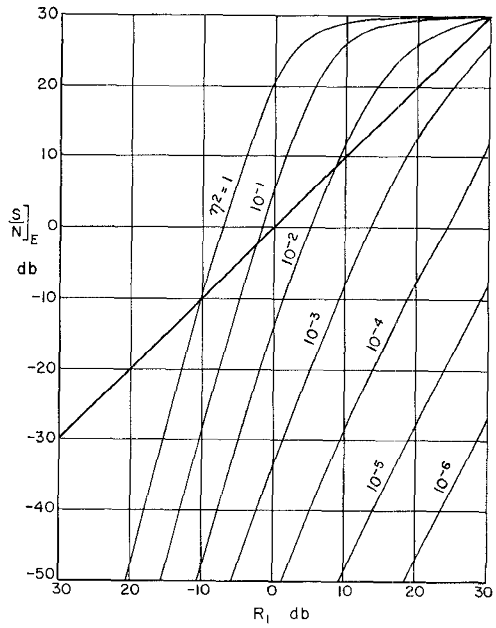

Fig. 6—The output SNR for the ENV type of correlation radiometer as a function of the receiver input SNR ($\beta=10^3$).

Tables I and II show the minimum detectable temperature or temperature increment for various cases of large and small input SNR's. Table I applies to the IF system and Table II to the ENV type of system.

The Effects of the Variation of Receiver Paramcters upon the Radiometer Sensitivity

Thus far only the ideal case has been considered where the gain and phase characteristics of the receiver have been assumed to remain constant. However, since any change in the receiver characteristics will cause a subsequent variation in the output signal which cannot be distinguished from the variations caused by the input signal, such changes must be taken into account when calculating the over-all sensitivity of the radiometer.

Gain fluctuations: Let the gain of the receiver be given by

$$A_{j(t)} = A_{0j} + \Delta A_{j(t)}, \tag{22}$$

where $\Delta A_j(t)$ is the small variation in the receiver gain. Then in the IF system the input to the correlator becomes

$$U_{j(t)} = A_{0j}(1 + \Delta A_{j(t)}/A_{0j})(s_{j(t)} + n_{j(t)}). \tag{23}$$

Carrying through the development as before, we find that the power SNR at the input to the correlator is now

$$R_{\text{in},f,j} = \frac{\psi_{sj}}{\psi_{nj} + a_j(\psi_{sj} + \psi_{nj})} = \frac{R_{\text{in},j}}{1 + a_j(1 + R_{\text{in},j})}, \tag{24}$$

where $\psi_{s1}=\psi_s$, $\psi_{s2}=\eta^2\psi_s$, $a_j=\psi_{Aj}/A_{0j}^2$, $\overline{\Delta A}=0$, $\tau=0$, and ψ_{Aj} is the mean-squared value of the gain fluctuations. The subscript f indicates that the gain fluctuation effects are included. Using (24) we find the expression for the ouptut SNR in the case of the IF type of system to be

TABLE I
IF TYPE*

	SNR_{input}	SNR_{output}	T_{min} or	ΔT	Remarks
1	$R_1 \gg 1$ $\eta^2 R_2 \gg 1$	α		$\frac{T_1}{2\alpha}$	$\frac{S}{N}\Big]_I \gg 1$
2	$R_1 \gg 1$ $\eta^2 R_2 \doteqdot 1$	$\frac{2}{3}\alpha$		$\frac{3}{4}\frac{T_1}{\alpha}$	$\frac{S}{N}\Big]_I \gg 1$
3	$R_1 \gg 1$ $\eta^2 R_2 \ll 1$	$2\eta^2 R_2 \alpha$	$\frac{F_e T_0}{2\alpha}$		$\frac{S}{N}\Big]_I \leq 1$
				$\frac{T_n}{4\alpha\eta^2}$	$\frac{S}{N}\Big]_I \gg 1$
	$R_1 \doteqdot 1$ $\eta^2 R_2 \ll 1$	$\eta^2 R_2 \alpha$	$\frac{F_e T_0}{\alpha}$		$\frac{S}{N}\Big]_I \leq 1$
				$\frac{T_n}{2\alpha\eta^2}$	$\frac{S}{N}\Big]_I \gg 1$
4	$R_1 \ll 1$ $\eta^2 R_2 \ll 1$	$2\eta^2 R_1 R_2 \alpha$	(1) $\frac{F_e T_0}{\sqrt{2\alpha}}$ $T_{n1} = T_{n2}$ (2) $\frac{T_0\sqrt{F_{e1}F_{e2}}}{\sqrt{2\alpha}}$ $T_{n1} \neq T_{n2}$		$\frac{S}{N}\Big]_I \leq 1$ $\eta_2 R_2 = R_1$

* (a) $T_{n1}=T_{n2}=T_n$ and $F_{e1}=F_{e2}=F_e$ have been assumed, unless otherwise indicated. (b) $T_{nj}=F_{ej}T_0$.

TABLE II
ENV TYPE*

	SNR_{input}	SNR_{output}	T_{min} or	ΔT	Remarks
1	$R_1 \gg 1$ $\eta^2 R_2 \gg 1$			$\frac{T_1}{4\beta}$	$\frac{S}{N}\Big]_E \gg 1$
2	$R_1 \gg 1$ $\eta^2 R_2 \doteqdot 1$	$\frac{2}{5}\beta$		$\frac{5}{8}\frac{T_1}{\beta}$	$\frac{S}{N}\Big]_E \gg 1$
3	$R_1 \gg 1$ $\eta^2 R_2 \ll 1$	$2\eta^4 R_2^2 \beta$	$\frac{F_e T_0}{\sqrt{2\beta}}$		$\frac{S}{N}\Big]_E \leq 1$
				$\frac{T_n^2}{8\beta\eta^4}$	$\frac{S}{N}\Big]_E \gg 1$
	$R_1 \doteqdot 1$ $\eta^2 R_2 \ll 1$	$\frac{1}{2}\eta^4 R_2^2 \beta$	$\sqrt{2}\cdot\frac{F_e T_0}{\sqrt{\beta}}$		$\frac{S}{N}\Big]_E \leq 1$
				$\frac{T_n^2}{2\beta\eta^4}$	$\frac{S}{N}\Big]_E \gg 1$
4	$R_1 \ll 1$ $\eta^2 R_2 \ll 1$	$2\eta^4 R_1^2 R_2^2 \beta$	(1) $\frac{F_e T_0}{(2\beta)^{1/4}}$ $T_{n1} = T_{n2}$ (2) $\frac{T_0\sqrt{F_{e1}F_{e2}}}{(2\beta)^{1/4}}$ $T_{n1} \neq T_{n2}$		$\frac{S}{N}\Big]_E \leq 1$ $\eta^2 R_2 = R_1$

* (a) $T_{n1}=T_{n2}=T_n$ and $F_{e1}=F_{e2}=F_e$ have been assumed, unless otherwise indicated. (b) $T_{nj}=F_{ej}T_0$.

$$\left.\frac{S}{N}\right]_{f,I} = \frac{2(R_{\text{in},1})(R_{\text{in},2})}{(R_{\text{in},1})(R_{\text{in},2}) + (R_{\text{in},1}+1)(R_{\text{in},2}+1)(1+\Gamma_I)}\cdot\alpha, \quad (25)$$

where $\Gamma_I \doteqdot a_1+a_2+a_1a_2$. For the case where the $T_{\min}$ is limited by $[S/N]_I=1$, and both $R_{\text{in},1}$ and $R_{\text{in},2}$ are much less than unity, we find that

$$T_{\min}]_{fI} \doteqdot \left(\frac{T_n}{\sqrt{2\alpha}}\right)\left(1+\frac{1}{2}\sum_{j=1}^{2}\frac{\psi_{Aj}}{A_{0j}^2}\right), \quad (26)$$

where T_n is defined in Tables I and II.

For the case on the ENV type of system, the SNR at the correlator input is given by

$$R'_{\text{in},f,j} = \frac{\psi_{sj}^2}{2\psi_{sj}\psi_{nj}+\psi_{nj}^2+b_j(\psi_{sj}+\psi_{nj})^2} \quad (27a)$$

$$= \frac{(R_{\text{in},j})^2}{2(R_{\text{in},j})+1+b_j(R_{\text{in},j}+1)^2}, \quad (27b)$$

where $b_j=8\psi_{Aj}/A_{0j}^2$, when $\tau=0$. We then find that the output SNR, taking gain variations into account, is given by

$$\left.\frac{S}{N}\right]_{f,E} = \frac{2(R_{\text{in},1})^2(R_{\text{in},2})^2}{(R_{\text{in},1})^2(R_{\text{in},2})^2+(R_{\text{in},1}+1)^2(R_{\text{in},2}+1)^2(1+\Gamma_E)}\cdot\beta, \quad (28)$$

where $\Gamma_E \doteqdot b_1+b_2+b_1b_2$. For the case when the $T_{\min}$ is limited by $[S/N]_E=1$,

$$T_{\min}]_{fE} \doteqdot [T_n/(2\beta)^{1/4}]\cdot\left[1+\frac{1}{4}\sum_{j=1}^{2}\frac{8\psi_{Aj}}{A_{0j}^2}\right]. \quad (29)$$

From the above it may be seen that if the same receivers are used in the IF and ENV systems, then the ENV system would have somewhat larger apparent variations in $T_{\min}$ due to gain variations than would the IF type of system. This occurs since $\Gamma_E \doteqdot 8\Gamma_I$ and $(2\beta)^{-\frac{1}{4}} > (2\alpha)^{-\frac{1}{2}}$. In the case of the unity output SNR, the above shows that gain fluctuations are not very important. In the case of a large signal input, a variation in the output which is proportional to the input signal is observed. Thus, in this case the gain fluctuations become very important in the correlation radiometer, since unlike the Dicke radiometer there is no matched load which may be adjusted in temperature to minimize the effects of the receiver gain variations.

The effects of receiver phase shift variations: In the IF type of system, phase fluctuations in the receivers would introduce errors into the output signal, since any variations in the phase characteristics of either one or the other of the receivers would cause at least some incoherence between the two receiver outputs. This degradation of the coherence may be treated as a noise source. In the ENV system, the phase shift of the received signal itself will not introduce any errors in the output, since all of this phase information is lost in the square-law detector anyhow. However, any shift in the phase of the signal *envelope* will cause output errors.

Consider that a phase or time fluctuation, x_θ, is introduced into channel 2 just prior to the correlator. Then

$$U_1 = U_1(t), \quad (30)$$

$$U_2 = U_2(t+\theta_r+x_\theta), \quad (31)$$

where θ_r is the mean value of the channel 2 phase shift and x_θ is the variation in the phase shift. If it can be assumed that x_θ has a Gaussian distribution, then the output autocorrelation function of the correlator is expressed as

$$\phi_m(\tau,\theta) = x_\theta\overline{\langle U_1(t)U_2(t+\theta+x_\theta)U_1(t+\tau)U_2(t+\theta+x_\theta+\tau)\rangle} \quad (32)$$

$$= \int \overline{U_1U_2U_{1\tau}U_{2\tau}}\,P(x_\theta)dx_\theta,$$

where $\langle\ \rangle$ denotes the ensemble average over x_θ, and $P(x_\theta)$ is the probability distribution of x_θ which is expressed by

$$P(x_\theta) = \frac{1}{\sqrt{2\pi}\sigma_\theta}\epsilon^{-x_\theta^2/2\sigma_\theta^2}, \quad (33)$$

where σ_θ^2 = the variance of x_θ. Usually x_θ is assumed to have the properties of stationarity and ergodicity. Using the above, one can obtain (see Appendix I)

$$\left.\frac{S}{N}\right]_I = \frac{\eta^2\psi_s^2(1-\sigma_\theta^2\omega_0^2)}{\frac{\omega_L}{2(\Delta\omega_{\text{IF}})}F(\psi_s,\psi_{nj})+\sigma_\theta^2\eta^2\psi_s^2\frac{\omega_L(\Delta\omega_{\text{IF}})}{4}} \quad (34)$$

for the output SNR. Here $F(\psi_s, \psi_{nj})$ stands for the denominator of the expression in (10).

From the above it can be seen that if σ_θ is large, considerable degradation of the SNR will result from the increase in the noise level and the simultaneous decrease in the signal level. If $\sigma_\theta^2\omega_0^2$ approaches unity, the SNR would fall to a very small value. Thus, for a good SNR, σ_θ should be much less than $1/\omega_0$.

Comparison with the Dicke Radiometer

The special characteristics of the correlation radiometer, both favorable and unfavorable in comparison with the Dicke radiometer, have been discussed qualitatively in the Introduction. Some of the characteristics of the correlation-type radiometer will now be compared with those of the Dicke radiometer on a quantitative basis.

In the case where the input signals to both channels of the correlation radiometer are small as compared to the noise signals in each channel, the minimum detectable temperature of the IF type of correlation radiometer can be shown to be (when the gain fluctuations are included)

$$T_{\min}]_{fI} = \kappa_I\left[\frac{T_n}{\sqrt{\alpha}}\left(1 + \frac{1}{2}\sum_{j=1}^{2}\frac{\psi_{Aj}}{A_{0j}^2}\right]\right), \qquad (35)$$

where κ_I is a constant. In the following, D will denote the Dicke radiometer, I the IF type radiometer, and E the ENV type radiometer. The sensitivity of the Dicke radiometer is given by

$$T_{\min}]_{fD} = \kappa_D\left[\frac{T_n}{\sqrt{\alpha}} + \frac{\Delta G}{G_0}(\Delta T_A + T_s)\right], \qquad (36)$$

where κ_D is another constant, T_n is the equivalent system noise temperature, $\Delta G/G_0$ is the gain fluctuation factor, G_0 is the mean value of the gain, ΔT_A is the temperature difference between the source and the load temperatures and T_s is the noise temperature of the source. It can be seen that the first terms in (35) and (36) can be regarded as being similar terms. The second terms are also comparable, since the term

$$\frac{1}{2}\sum_{j=1}^{2}\frac{\psi_{Aj}}{A_{0j}^2}$$

may correspond to $\Delta G/G_0$, and the terms $T_n/\sqrt{2}$ and $(\Delta T_A + T_s)$ can be considered to be more or less equivalent. As was shown in the preceding sections, the ENV type of radiometer had a poorer sensitivity than the IF type of radiometer. It can be shown that the ENV type of system is poorer than the IF type of system by a factor of $(2)^{-\frac{1}{4}}(\alpha)^{-\frac{1}{2}}(\beta)^{\frac{1}{4}}$.

In the case where one or the other of the signal inputs is no longer small, *e.g.*, $R_1 \gg 1 \gg \eta^2 R_2$, it can be shown that the minimum detectable temperatures for the case where the output SNR$\gg$1 is

$$\Delta T_I = \frac{T_n}{4\alpha\eta^2} \qquad \text{(IF type)} \qquad (T_n = F_e T_0), \qquad (37)$$

$$\Delta T_E = \frac{T_n^2}{8\beta\eta^4} \qquad \text{(ENV type)} \qquad (T_n = F_e T_0), \qquad (38)$$

which shows that the sensitivity depends upon T_n and η.

If we compare the sensitivity of the above systems with a Dicke system looking at the larger source, we find that for the Dicke system

$$\Delta T_D = \kappa_D \frac{T_1}{\alpha}, \qquad (39)$$

where F_e is the equivalent noise figure of the receivers in all cases, and T_0 is the standard temperature (commonly taken to be 290°K). The latter relation can be derived from the equation for the output SNR in Goldstein's paper [3]. Comparing the above, it is seen that the correlation radiometer would have a lower sensitivity than the Dicke radiometer because $(\eta^2 T_1/T_n) < 1$.

For the case where the output SNR is close to unity, $T_{\min}$ is given by

$$T_{\min}]_I = \frac{F_e T_0}{2\alpha} \qquad \text{(IF type)}, \qquad (40)$$

$$T_{\min}]_E = \frac{F_e T_0}{\sqrt{2\beta}} \qquad \text{(ENV type)}. \qquad (41)$$

Since the sensitivity in the Dicke case is expressed by

$$T_{\min}]_D = \kappa_D \frac{F_e T_0}{\sqrt{\alpha}}, \qquad (42)$$

we see that the IF type of radiometer would be more sensitive than the Dicke radiometer, in this case, by a factor of $\sqrt{\alpha}$, while the ENV type of radiometer would have a sensitivity on the same order of magnitude as the Dicke radiometer except for factor of order unity due to the constant term.

Conclusions

The two basic types of correlation radiometer, the IF type and the envelope-detection type (ENV type), have been discussed and compared. The SNR at the output and the minimum detectable temperature increments have been expressed in terms of the SNR's at the inputs. These results have shown that the IF type of radiometer is superior to the ENV type of system in terms of the sensitivity, and should be used except where the phase information of the two input signals is uncorrelated. The effects of gain fluctuations upon the minimum detectable temperature have been compared for the IF, the ENV and the Dicke types of systems in the weak-signal case. It was found that the IF and the Dicke systems gave comparable results, but the effect of gain fluctuations in the ENV type of system was worse than in the IF type of system. It was also shown that phase fluctuations in the receivers could result in large degradations in the system sensitivity.

The most useful applications of the correlation-type radiometers would be in the millimeter or submillimeter wavelength regions, where the elimination of the microwave switch which is used in the Dicke system is the main advantage which the correlation radiometer can claim; in interferometer systems (IF type); and for the studying of the correlation (ENV type of system) between the signals received from two different sources, such as the sun and the moon.

Appendix I

Evaluation of the Effects of the Receiver Phase Fluctuations

Starting with the expressions for the inputs to the correlator

$$U_1(t) = s(t) + n_1(t), \qquad (43)$$

$$U_2(t) = \eta s(t + \theta + x_\theta) + n_2(t + \theta + x_\theta), \qquad (44)$$

the correlation function of the correlator output is found to be

$$\phi_m = \int \left[\eta^2\{\phi_s^2(\theta + x_\theta) + \phi_s^2(\tau) + \phi_s(\theta + \tau + x_\theta)\phi_s(\theta + x_\theta - \tau)\} + \phi_s(\tau)\phi_{n2}(\tau) + \eta^2\phi_s(\tau)\phi_{n1}(\tau) + \phi_{n1}(\tau)\phi_{n2}(\tau)\right] P(x_\theta)dx_\theta. \quad (45)$$

Expanding the above in a Taylor Series about $\theta = \theta_r$, the following relations are obtained for the τ-independent terms and the τ-dependent terms, respectively:

$$\eta^2\psi_s^2\epsilon^{-\Delta\omega_{\mathrm{IF}}\theta}[\cos^2 \omega_0\theta - \sigma_\theta^2\omega_0^2 \cos 2\omega_0\theta], \quad (46)$$

$$F_{(\psi_s,\psi_{nj})} \frac{\omega_L}{2\Delta\omega_{\mathrm{IF}}} + \sigma_\theta^2\eta^2\psi_s^2 \frac{\omega_L\Delta\omega_{\mathrm{IF}}}{4} \cdot \left[\frac{\Delta\omega_{\mathrm{IF}}}{\omega_L}(1 - \epsilon^{-2\omega_L\theta}) + 1\right]\epsilon^{-\Delta\omega_{\mathrm{IF}}\theta} \cos 2\omega_0\theta. \quad (47)$$

For the case of $\theta = 0$ this leads to the following relation for the output SNR:

$$\left.\frac{S}{N}\right]_I = \frac{\eta^2\psi_s^2(1 - \sigma_\theta^2\omega_0^2)}{F_{(\psi_s,\psi_{nj})} \frac{\omega_L}{2\Delta\omega_{\mathrm{IF}}} + \sigma_\theta^2\eta^2\psi_s^2 \frac{\omega_L\Delta\omega_{\mathrm{IF}}}{4}}. \quad (48)$$

(In the above calculations it has been assumed that $\omega_s = \omega_{n1} = \omega_{n2} = \Delta\omega_{\mathrm{IF}}/2 \ll \omega_0$, and the terms above the second order have been omitted.)

Appendix II

Glossary of Symbols

s_j = input signal voltage
n_j = input noise voltage
ψ_{sj} = mean-square value of input signal voltage
ψ_{nj} = mean-square value of input noise voltage
U_j = voltage input to the correlator (IF system)
t = time
θ = time delay
$W_{(t)}$ = output voltage of multiplier (IF system)
τ, τ' = autocorrelation and convolution time factor
$\phi_m(\tau, \theta)$ = autocorrelation function of $W_{(t)}$
$\phi_0(\tau, \theta)$ = autocorrelation function of $W_{(t)}$ after being put through a low-pass filter
$H_{(\omega)}$ = transfer function of the low-pass filter
ω = angular frequency
S/N = power SNR at correlator output
(ΔT) = minimum detectable source temperature change
$T_{\min}$ = minimum detectable source temperature
Y_j = input voltage to correlator (ENV system)
$A_{j(t)}$ = voltage gain of microwave receiver
A_{0j} = mean voltage gain of microwave receiver
$\Delta A_{j(t)}$ = variation in the voltage gain of microwave receiver
ψ_{Aj} = mean-squared value of receiver gain fluctuations
η = amplitude ratio between s_1 and s_2
ω_s, ω_n = effective $\frac{1}{2}$ bandwidth of signal and noise, respectively
ω_L = cutoff frequency of low-pass filter (integrator)
$\Delta\omega_d$ = bandwidth immediately preceding correlator (ENV system)
ω_0 = IF amplifier center frequency
$\Delta\omega_{\mathrm{IF}}$ = IF amplifier bandwidth
$R_{\mathrm{in},j}$ = power SNR at correlator input (IF system)
R_i = effective power SNR at receiver input
$R_{I,E}$ = power SNR at output
$\alpha = \Delta\omega_{\mathrm{IF}}/\omega_L$
$\beta = \omega_d/\omega_L$
$R'_{\mathrm{in},j}$ = power SNR at correlator input (ENV system)
c_I, c_E = arbitrary constants
S = squared value of correlator dc output
ξ = 2 (IF systems)
= 4 (ENV systems)
$\Gamma_I \doteq a_1 + a_2 + a_1a_2$ where $a_j = \psi_{Aj}/(A_{0j})^2$
$\Gamma_E \doteq b_1 + b_2 + b_1b_2$ where $b_j = 8\psi_{Aj}/(A_{0j})^2$
θ_r = mean value of θ
x_θ = variation in θ
$P(x_\theta)$ = probability distribution of x_θ
σ_θ = standard deviation of x_θ about θ_r
κ_I, κ_D = constants taking into account recorder noise, reading error, etc.
T_n = effective receiver noise temperature
ΔT_A = difference between source and load temperatures (Dicke system)
T_s = source temperature (Dicke system)
ΔG = gain variation (Dicke system)
G_0 = mean gain (Dicke system)
F_e = equivalent receiver noise figure

Subscripts:

j = refers to channel (1 or 2)
I = refers to IF system
E = refers to ENV system
D = refers to Dicke system
in = refers to correlator input
f = means that amplifier fluctuation factor is included

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Some Aspects of Beam Waveguides for Long Distance Transmission at Optical Frequencies

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Summary—Two types of beam waveguides are discussed in this paper, the iris-type and the lens-type. Both appear applicable to guided long distance transmission of light with theoretical losses of less than 1 db/km. However, there are problems concerning their practicability which require experimental investigation. Such problems are the alignment of the irises or the lenses, the effects of turbulence and stratification of air along the light path, and the required tolerance in the construction of the lenses. Since the lens-type guide offers a simple possibility for compensating misalignments, an experimental waveguide of this type has been constructed, having a length of approximately 1 km and comprising 10 iterations. The light path is enclosed by a 4 inch aluminum pipe which is supported within a 6 inch aluminum pipe. The first series of experiments which is reported in this paper indicated that there are no serious alignment problems. However, it was found that the effects of turbulence and air stratification are usually very severe and it appears necessary to provide an evacuated light path to obtain constant transmission conditions. It was also found that the available lenses add considerably higher iteration loss than expected. This increased loss was primarily caused by inadequate surface coating. A theoretical study of beam propagation in a misaligned lens-type guide is included in the Appendix.

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INTRODUCTION

WITH THE RECENT development of the optical maser (laser), an extremely large frequency range has been made available to communications. Unfortunately, the utilization of this frequency range is seriously hampered by the vulnerability of light propagation through the atmosphere not only by fog, rain or snow, but also by turbulence. An obvious way to overcome this difficulty is to provide a protected light path. Efficient long-distance transmission would, however, not result if the light beam were simply passed through an ordinary pipe. Although it is possible to produce coherent optical beams of extremely small divergence, it would be quite expensive to provide pipelines which were so straight that the beam would not hit the wall within a distance of a few hundred meters. Reflections on the wall of the pipe would not only cause substantial transmission loss, but also severe delay distortions. Eaglesfield [1] discusses the possibility of transmitting light through a pipe of precision bore whose inner surface has a mirror finish. In this case, light is propagated by multiple internal reflections. Although the theoretical loss is quite small, there are in-